

Efficient Methodology of Roll Load Prediction on 2D bodies in Nonlinear Flows

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1 INTRODUCTION

The response of a structure, be it freely floating or constrained, to a sea state is the most interesting aspect for naval architects as it dictates the survivability and habitability of the structure. This calls for the accurate estimation of the generalized loads on the structure. In the standard setting, the pressure is integrated along the wetted surface to find the generalized loads. But, the Bernoulli pressure requires the evaluation of the temporal and the spatial gradients of the velocity potential on the body, which are cumbersome. P.D.Sclavounos(P.D.Sclavounos [2012]) has recently applied the Reynolds Transport Theorem to the fluid momentum on a closed domain for yielding generalized forces on structure. But, he assumes that there is no contribution of the control surfaces at infinity, since the disturbance due to the presence of the body has a dipole like decay. This is possible in three dimensional flows, due to the direction spreading of the energy.

P.D.Sclavounos and S.Lee(P.D.Sclavounos and S.Lee [2013]) apply the Fluid Impulse Theory for weak scatter flows, where they employ Havelock Greens functions and find the sectional loads on a ship using 2.5D+T theory. But the weak scatter approximation, though is physically justifiable for slender ships, is mathematically inconsistent. Thus, in order to consider complete nonlinear flows, a re-derivation of the Fluid Impulse Theory was carried following along the lines of P.D.Sclavounos. In section 2, the derivation of the re-formulated Fluid Impulse Theory is detailed. We apply and compare the results of the re-formulated Fluid Impulse Theory with the standard Direct Pressure Integration in section 3 to the complete nonlinear wave-body interaction problem, by dividing into two segments: a nonlinear diffraction program, where the body is fixed and nonlinear wave impinges on it and the nonlinear radiation program, where the body is forced to undergo roll motion with a prescribed amplitude at a prescribed frequency in initially calm water. Section 4 compares the advantages and disadvantages of the Fluid Impulse Theory with the Direct Pressure Integration.

2 RE-FORMULATED FLUID IMPULSE THEORY

Before, we start by providing the derivation of the Fluid Impulse Theory, we assume the fluid is irrotational, inviscid and incompressible. An Eulerian description of flow is employed in this work.

Denoting $V(t)$ as the volume bounded by the computational domain(refer Fig 1), whose boundaries are given by $S(t)$ which include the lateral boundaries(upstream(S_{UP}) and downstream(S_{DS})), Free surface(left hand side(S_{FL}) and right hand side(S_{FR}) of the body), the body(S_B) and the bottom(S_{BO}). We do not specify any boundary conditions in the beginning and go about with no assumptions, until stated. By virtue of Reynold Transport Theorem applied to the fluid momentum in our closed domain:

$$\frac{D}{Dt} \int_{V(t)} \nabla \phi dv = \int_{V(t)} \frac{\partial \nabla \phi}{\partial t} dv + \int_{S(t)} \nabla \phi (U \cdot \hat{n}) ds \quad (1)$$

Applying the Gauss theorem on the left hand side and on the first term on the right hand side, after swapping the time derivative with the gradient of the potential, we have:

$$\frac{D}{Dt} \int_{S(t)} \phi \hat{n} ds = \int_{S(t)} \left[\frac{\partial \phi}{\partial t} \hat{n} + \nabla \phi (U \cdot \hat{n}) \right] ds \quad (2)$$

As stated, we employ an Eulerian representation of the free surface(that is the points are only allowed to move in Y-direction and frozen from the X- motion). The consequence of this representation is that the normal velocity of the free surface(in our representation) is not equal to the normal gradient of the

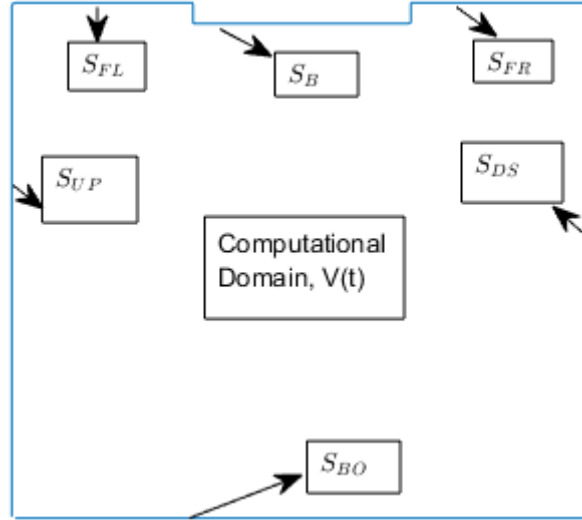


Figure 1: Computational Volume along with various surfaces

velocity potential on the free surface. So, again let us add and subtract the term $\int_{S_{FSL}+S_{FSR}} \nabla \phi \frac{\partial \phi}{\partial n} ds$ to the derivation and assume that the velocity of the free surface is U for the time being (later we replace by vertical gradient of the potential).

$$\frac{D}{Dt} \int_{S(t)} \phi \hat{n} ds = \int_{S(t)} \left[\left(\frac{\partial \phi}{\partial t} \hat{n} + \nabla \phi \frac{\partial \phi}{\partial n} \right) \right] ds - \int_{S_{UP}+S_{DS}+S_{FL}+S_{FR}} \nabla \phi \frac{\partial \phi}{\partial n} ds + \int_{S_{FL}+S_{FR}} \nabla \phi (U \cdot \hat{n}) ds \quad (3)$$

Let us deal with the whole derivation, by assuming that the last two terms are not there and latter lets add them. After re-arranging, this will lead to:

$$\int_{S(t)} \frac{\partial \phi}{\partial t} \hat{n} ds = \frac{D}{Dt} \int_{S(t)} \phi \hat{n} ds - \int_{S(t)} \nabla \phi \frac{\partial \phi}{\partial n} ds \quad (4)$$

Now adding $\int_{S(t)} \frac{1}{2} \nabla \phi \cdot \nabla \phi \hat{n} ds$ and $\int_{S(t)} gZ \hat{n} ds$ on to both sides of the expression above leads us to:

$$\int_{S(t)} \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gZ \right] \hat{n} ds = \frac{D}{Dt} \int_{S(t)} \phi \hat{n} ds - \int_{S(t)} \left[\nabla \phi \frac{\partial \phi}{\partial n} - \frac{1}{2} \nabla \phi \cdot \nabla \phi \hat{n} \right] ds + \int_{S(t)} gZ \hat{n} ds \quad (5)$$

For any closed volume, the following identity holds:

$$\int_{S(t)} \left[\nabla \phi \frac{\partial \phi}{\partial n} - \frac{1}{2} \nabla \phi \cdot \nabla \phi \hat{n} \right] ds = 0 \quad (6)$$

This is the reason for adding terms to (2). Substituting (6), leads to simpler expression:

$$\int_{S(t)} \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + gZ \right] \hat{n} ds = \frac{D}{Dt} \int_{S(t)} \phi \hat{n} ds + \int_{S(t)} gZ \hat{n} ds \quad (7)$$

We want to be more specific about the details, so we deal with the left hand side and right hand side of the expression independently. We first go about with the left hand side of the expression. If we break the expression into the integrals over the surface, we see that we are simply left with expressions of the pressure. This was initially the reason of adding terms (in eq 4) on the both sides of the equation. We recollect that the pressure on the free surface should be identically zero. Representing the integral of pressure over the wetted surface of the body as negative of the force exerted on the body (F_B), we finally end up with: Re-arranging the terms and incorporating those additional terms which we skipped leads to the final expression (8).

$$F_B = \int_{S_{UP}+S_{DS}+S_{BO}} p \hat{n} ds - \rho \frac{D}{Dt} \int_{S(t)} \phi \hat{n} ds - \rho \int_{S(t)} gZ \hat{n} ds - \rho \int_{S_{UP}+S_{DS}+S_{FL}+S_{FR}} \nabla \phi \frac{\partial \phi}{\partial n} ds + \rho \int_{S_{FSL}+S_{FSR}} \nabla \phi (U \cdot \hat{n}) ds \quad (8)$$

In the expression (8), we can see that the total generalized force on the structure is composed of the pressure loads from the upstream, downstream and the bottom boundary, time derivative of impulses on all the boundaries of computational domain, the body force and additional terms that arise due to

Variable	Value
Length of the computational domain	60m
Depth of the computational domain	150m
Beam of the barge	12m
Draft of the barge	2.4m
Wave height	0.1m
Wavelength	12m
Order of the wave	5

Table 1: Parameters for the nonlinear diffraction program

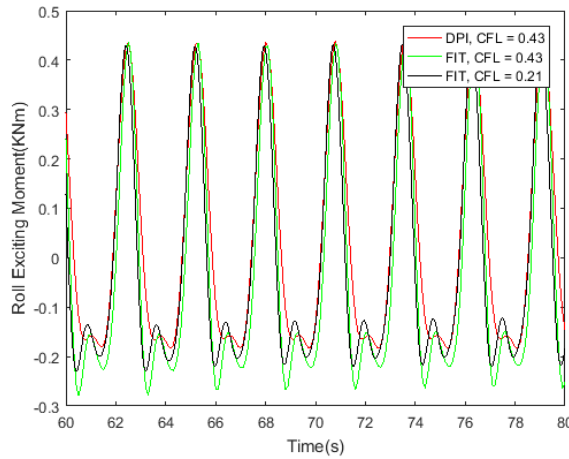


Figure 2: Roll exciting moments on barge of beam $B=12\text{m}$ and draft $T=2.4\text{m}$ by waves of waveheight of 0.1m and the wavelength 12m

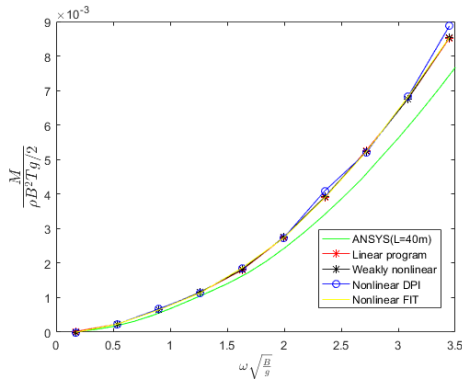
the Eulerian representation of the flow.

3 RESULTS

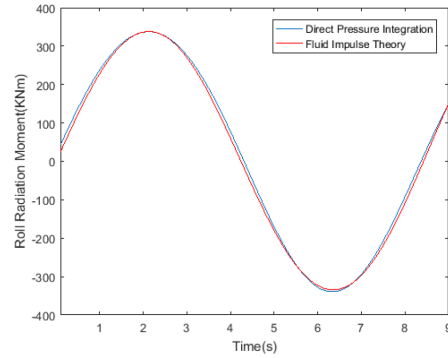
We first provide the results of the nonlinear diffraction program and follow up, it with nonlinear radiation program. The parameters that serve as an input to the nonlinear diffraction program is provided in Table 1. We employ the incident wave potential from the Rienecker and Fenton algorithm (M.M.Rienecker and J.D.Fenton [1981]). We choose a condition, where the wavelength of the impinging wave equals the ship beam. In Fig 2, we provide time trace of the roll exciting moment using Direct Pressure Integration and the reformulated Fluid Impulse Theory. By taking the Fourier Transform of the sufficiently long roll exciting moment time trace, we get the frequency content which are tabulated in Table 2. As we can see in Fig 2, the reformulated Fluid Impulse Theory requires a lower Courant Friedrich Lewy (CFL) number (in this case, half of the value employed for standard Direct Pressure Integration) for achieving the same accuracy as using standard Direct Pressure Integration.

Order	Wave steepness	$\frac{B}{\lambda}$	Technique	$\frac{M}{\rho B^2 T \frac{g}{2} \frac{H}{\lambda}}$		
				Mean	Linear	Sum
5	0.00833	1	DPI	0.0112	0.01055	0.0119
5	0.00833	1	FIT	0.0102	0.0106	0.017532

Table 2: Comparison of the results of the standard Direct Pressure Integration (DPI) and the reformulated Fluid Impulse Theory (FIT), where Linear implies the contribution to the roll exciting moment at wave frequency, sum implies the contribution to the roll exciting moment at twice the wave frequency



(a) Comparison between the various radiation programs for the parameters provided in Table 3



(b) Comparison of Fluid Impulse Theory(FIT) and the Direct Pressure Integration(DPI) results for roll amplitude of 0.05rads at roll natural frequency(0.76rad/s)

Figure 3: Roll exciting moments on barge of beam $B=12\text{m}$ and draft $T=2.4\text{m}$ by waves of wave height of 0.1m and the wavelength 12m

For the nonlinear radiation program, we run simulations for wave frequencies close to the natural frequency of roll(0.76rad/s) and we choose the roll amplitude as 0.005 rads(fig 3(a)). For verification purposes, we provide the results of our linear program. Also, we provide a roll radiation moment time trace using the Direct Pressure Integration(DPI) and the re-formulated Fluid Impulse Theory(FIT) for roll amplitude of 0.05 rads at the roll natural frequency(0.76rad/s) in fig 3(b). This completes a comparison between the two different techniques, which are quite close.

4 CONCLUSIONS

In Direct Pressure Integration, the pressure is integrated along the wetted surface. The estimation of pressure on the body requires evaluation of the gradients(both spatial and temporal) of the potential. The evaluation of the gradients is not straight forward. Using the reformulated Fluid Impulse Theory, the generalized loads on the structure are evaluated without estimating the pressure on the body. This makes the problem, computationally less intensive. The results of the Direct Pressure Integration are sensitive to the mesh size on the body, whereas the reformulated Fluid Impulse Theory requires an accurate representation of the free surface and the body. This may partly outweigh the advantage, but to obtain a conclusive statement, further research is to be carried out.

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