Study on Asymptotic Formula for Added Resistance in Short Waves

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1 INTRODUCTION

Faltinsen et al. (1980) derived an asymptotic formula for added resistance of ships in short waves. This formulation is based on the assumption that a ship has vertical side at the water plane and that the incident wavelength is small compared to the ship draught. Generally, this asymptotic formula shows fair agreement with experimental data for blunt ships at low forward speed, whereas the underestimation of results can be observed for slender ships at relatively high forward speeds. The effects of local steady velocity and the shape above still-water-level (SWL) have been reported as two main reasons for underestimation in the literature.

In this study, the original asymptotic formula is modified by introducing the following three effects: finite draught of ships, local steady velocity, and shape above SWL. The effect of each parameter has been investigated and compared to experimental results and other empirical formula of added resistance in short waves.

2 ORIGINAL ASYMPTOTIC FORMULA

A ship is considered as a stationary, vertically long cylinder, which has the same cross-section to the water plane area of the ship. The coordinate system (x, y, z) is defined at the center of the cylinder and x- and z- directions indicate longitudinally afterward and vertically upward directions of the ship, respectively. The ship is moving forward with a constant speed U and the incident wave is approaching with wave amplitude ζ_a and circular frequency ω_0. The local coordinate system (n, s) along the water plane area curve can be defined, where n and s are orthogonal and tangential to the water plane area curve, respectively. Stretching the coordinate by the scale of the wavelength, the cylinder can be considered a vertical wall which has angle θ with the x-direction. If the ship speed is small, there is a horizontal steady velocity V parallel to the wall.

The flow is assumed as ideal flow. Using boundary conditions and the fictitious problem where the fictitious incident wave is assumed to be totally reflected, the total velocity potential Φ in many wavelengths away from the wall can be obtained as follows (Faltinsen et al., 1980):

$$\Phi = \frac{g\zeta_a}{\omega_0}\left[e^{k_0 z}\cos\left(k_0 s\cos\theta + k_0 n\sin\theta + k_0 x_0 - \omega_e t\right) + B_1 e^{k_1 z}\cos\left(k_0 s\cos\theta - k_2 n + k_0 x_0 - \omega_e t\right)\right] + Vs \tag{1}$$

where $k_1 = \dfrac{\left(\omega_e - Vk_0\cos\theta\right)^2}{g}, k_2 = \sqrt{k_1^2 - k_0^2\cos^2\theta}, \text{and } B_1 = \dfrac{2k_1}{k_1+k_0}\dfrac{k_0}{k_2}\sin\theta.$

Here, g the acceleration of gravity, k_0 the wave number, (x_0, y_0) the x, y-coordinates of the origin of the local coordinate system (n, s), ω_e the circular frequency of encounter, and t, the time variable. Based on the conservation of momentum, the sectional average force normal to the wall can be obtained as follows:

$$\Delta R_n = \frac{1}{2}\rho g\zeta_a^2\left[\frac{1}{2}\frac{k_1}{k_0} - \frac{1}{2}\cos^2\theta + \frac{1}{2}\frac{k_2}{k_0}\sin\theta\right]. \tag{2}$$

Using the assumption of local steady velocity $V = U\cos\theta$ and low advancing speed, the added resistance in short waves R_{ASW} can be finally obtained as follows:

$$\Delta R_n = \frac{1}{2}\rho g\zeta_a^2\left(1 + \frac{2\omega_0 U}{g}\right)\sin^2\theta \text{ and } R_{ASW} = \int_{WL}\Delta R_n n_1 dl \tag{3}$$

where n_1 is the sectional normal vector in the x-direction. The integration is performed over the non-shaded part WL of the water plane curve.

3 MODIFIED ASYMPTOTIC FORMULA

In many cases, the wavelength is longer than the draught of the ship even in short waves. However, it is difficult to calculate the diffraction wave potential for an arbitrary ship with finite draught because there will be transmitted waves. In this study, the horizontal control surface far below the free surface is assumed to be located at the level of ship draught, while the velocity potentials are the same as those given in the original derivation. Thus, the sectional force can be calculated based on the momentum conservation as follows:

$$R_{ASW} = \int_{WL} \Delta R_n n_1 dl = R_\zeta + R_{V\infty} + R_{VT}, \tag{4}$$

$$R_\zeta \equiv \frac{1}{2}\rho g \zeta_a^2 \int_{WL} \left(\frac{1}{2}\frac{k_1}{k_0}\right)\left[1 + B_1^2 + \frac{(k_0 + k_1)}{(k_2 - k\sin\theta)}\left(\sin\theta - B_1^2 \frac{k_2}{k_1}\right)\right] n_1 dl, \tag{5}$$

$$R_{V\infty} \equiv -\frac{1}{2}\rho g \zeta_a^2 \int_{WL} \left[\left(\frac{1}{2} + B_1^2 \frac{k_0}{2k_1}\right)\cos^2\theta + \frac{1}{2}\frac{\left(k_0\cos^2\theta + k_2\sin\theta + k_1\right)}{(k_2 - k_0\sin\theta)}\left(\sin\theta - B_1^2\frac{k_2}{k_1}\right)\right] n_1 dl, \tag{6}$$

$$R_{VT} \equiv \frac{1}{2}\rho g \zeta_a^2 \int_{WL} \frac{1}{2} \begin{bmatrix} \left(e^{-2k_0 T(x_0)} + B_1^2 \dfrac{k_0}{k_1} e^{-2k_1 T(x_0)}\right)\cos^2\theta \\ + e^{-(k_1+k_0)T(x_0)} \dfrac{\left(k_0\cos^2\theta + k_2\sin\theta + k_1\right)}{(k_2 - k_0\sin\theta)}\left(\sin\theta - B_1^2 \dfrac{k_2}{k_1}\right) \end{bmatrix} n_1 dl. \tag{7}$$

Here, $T(x_0)$ is the sectional draught of the ship and it will be equal to ship draught for even keel conditions. The first two terms R_ζ and $R_{V\infty}$ are the same as the original asymptotic formula and the additional term R_{VT} is included due to the integration of velocity square terms along the vertical control surface from the negative infinite to the ship draught.

The comparison of added resistance is shown in Figs. 1(a) and 2(a) for KVLCC2 with Froude number, $Fr = U/(gL)^{1/2}$ = 0.142, and for the S175 container ship with Fr = 0.2, respectively. In both cases, the correction effect due to finite draught is negligible for very short wavelengths ($\lambda/L < 0.2$), whereas the contribution of this term increases in the region of $\lambda/L > 0.2$. The results of the original asymptotic formula and the correction method due to finite draught indicated by 'Present ($+R_{VT}$)' in the legend lie within the range of experimental data (Lee et al., 2017) for the KVLCC2 case. In addition, the modified method provides similar results to those of the NMRI formula (Kuroda et al., 2008). On the other hand, the results are improved only for $\lambda/L > 0.5$ and the underestimation of added resistance in short waves is still obtained for S175 container ship compared to experimental data (Fujii and Takahashi, 1975; Nakamura and Naito, 1977).

It is reasonable to assume that the diffracted wave is dominantly influenced by the ship geometry only near SWL if the incident wave is short enough relative to the ship length and draught. However, the steady flow may vary with the bow shape below SWL in addition to the ships geometry near SWL. The proposed local steady velocity in the original derivation (Faltinsen et al., 1980) reflects only the geometrical characteristics at SWL and does not consider changes in shape below SWL. It is complicated to find the local steady velocity for general ship geometries. Therefore, a simple way to calculate the local steady velocity is required even when considering changes in bow shape below SWL.

The characteristics of steady flow around the bow of wedge-shaped bodies with flare and rake angles were studied in Noblesse et al. (2009). A conclusion from this study is that the maximum steady wave elevation is approximately proportional to the flare parameter. From Bernoulli's equation, it can be deduced that the local velocity is proportional to the square root of one minus flare parameter. In a similar way, the local steady velocity V is proposed in the following form keeping it as simple as possible:

$$V = \alpha_A(x_0) U\cos\theta = \min\left(1.0, \frac{2A(x_0)}{A_0(x_0) + A(x_0)}\right) U\cos\theta \tag{8}$$

where $A(x_0)=\int_{-T(x_0)}^{0} y dz$, and $A_0(x_0)=y_0 \times T(x_0)$. If the ship is a cylindrical shape with the same sectional area along the vertical direction, α_A becomes unity and eventually the local steady velocity of the original asymptotic formula is reproduced.

The modification of local steady velocity provides the increased added resistance of the S175 container ship especially for very short wavelengths ($\lambda/L < 0.2$), whereas almost the same results can be obtained for KVLCC2 as shown in Figs. 1 and 2. Here, the inclusion of the newly proposed local steady velocity is denoted by '$+V$' in the legend. Combining the corrections of finite draught and local steady velocity gives improved results of added resistance in the wavelength range considered in this study.

To take into account the variation of bow shape above SWL, the steady wave elevation is used as a baseline level. The maximum steady wave elevation η_0 for wedge-shaped bodies can be obtained as follows (Noblesse et al., 2008):

$$\frac{\eta_0 g}{U^2}=\min\left(\frac{2.2}{1+F_T}\frac{\tan\overline{\alpha_E}}{\cos\overline{\alpha_E}},\ 0.5\right) \text{ where } F_T=U/\sqrt{gT}. \tag{9}$$

Here, $\overline{\alpha_E}$ is half of the entrance angle. In this study, the bluntness coefficients B_f are initially calculated at $z = 0$ and $z = \eta_0/2$. Then, the ratio of bluntness coefficients at a different level, α_B, defined in the following form is multiplied by the added resistance in short waves calculated from the previous modified formula:

$$\alpha_B=\frac{B_f\left(z=\eta_0/2\right)}{B_f\left(z=0\right)} \text{ where } B_f=\frac{1}{B}\int_L \sin^3\theta dl. \tag{10}$$

Here, B is the ships breadth. The correction due to the bow shape above SWL shows almost identical result for the original KVLCC2 case, whereas the difference can be found in Ax-bow type hull as shown in Fig. 1. The Ax-bow type has a relatively sharp bow edge above SWL and it provides smaller added resistance than that of the original KVLCC2.

For the S175 container ship, α_B becomes larger than 1.0 because of the flare angle and consequently the added resistance increases. Moreover, α_B also depends on ship speed because the maximum steady elevation depends on the ship speed. Thus, the increase in added resistance due to α_B is larger in the faster case ($Fr = 0.25$) due to the positive flare angle of the S175 container ship.

4 CONCLUSIONS

In this study, practical correction methods have been incorporated into Faltinsen's asymptotic formula of added resistance in short waves in terms of these three aspects: finite draught, local steady velocity, and shape above SWL. The correction due to the finite draught of ships reduces the suction force and eventually the added resistance in short waves increases. As local steady velocity decreases, the added resistance in short waves increases. The newly proposed velocity, which includes changes of bow shape below SWL, provides the increased added resistance value for slender ships. To consider the variation of bow shape above SWL, a baseline level was calculated from the steady wave elevation and the ratio of bluntness coefficients between the SWL and the baseline level was multiplied to the added resistance in short waves. Using this approach, the effect of the change of bow shape above SWL on added resistance in short waves can be considered. The added resistance in the overall range of wavelengths by the combination of the proposed method and the existing computational results will be presented at the workshop.

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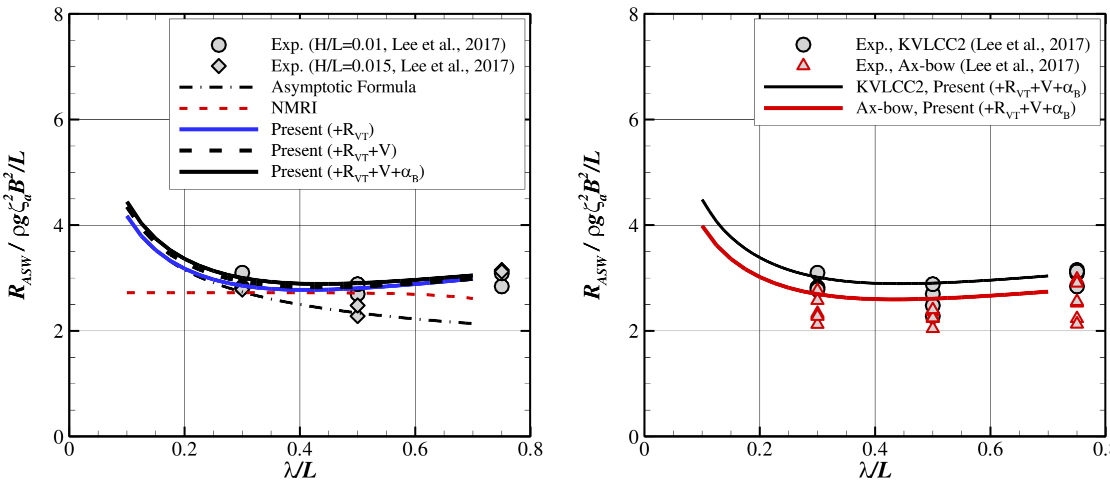
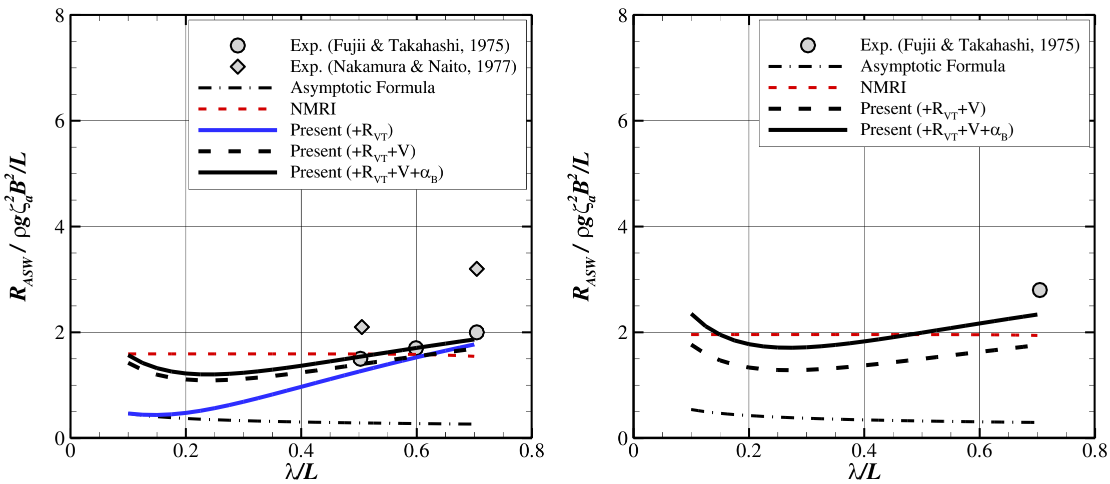


(a) KVLCC2 (b) KVLCC2 and Ax-bow type

Fig. 1 Added resistance of KVLCC2 and Ax-bow type in short waves, $Fr = 0.142$



(a) $Fr = 0.2$ (b) $Fr = 0.25$

Fig. 2 Added resistance of S175 container ship in short waves with different Froude number