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# Wave Transmission through Vertical Thin Barriers with Gaps in Channels by a Hyper Singular BEM

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## Highlights

1. A hyper singular BEM model is set up for wave diffraction from thin vertical barriers in channels.
2. Numerical examinations are carried out on extraordinary transmission of waves for channels with different channel widths and barrier arrangements.

## 1 Introduction

It is known that extraordinary transmission (ET) of waves can occur at some special frequencies in a channel with a group of uniformly arranged barriers with gaps. Evans and Porter (2015) developed a small gap approximation method and examined the total transmission of waves through 4 pairs of uniformly arranged barriers with narrow gaps in a narrow channel. This method is applicable when only one propagation mode exists in the channel.

For studying practical cases with different arrangements of barriers, a hyper singular BEM is developed for thin structures in channels in this study, which is applicable for wide channels and those with un-uniformly distributed barriers, as auxiliary propagation waves are included in the BEM model and in the computation of transmission and reflection wave energy fluxes. With the numerical model, examinations are carried out on the influence of wave transmission due to channel width, gap width, barrier interval, and barrier arrangement. The relationships of extraordinary transmission frequencies with those parameters are shown.

## 2 Formulations

Consider the wave transmission through a channel of width $d$, in which some vertical barriers are arranged at $X_j$ transversely, which have gaps of width $a$ between them and a side of the channel, as Fig. 1. This is the same with the symmetric problem studied by Evans and Porter (2015).

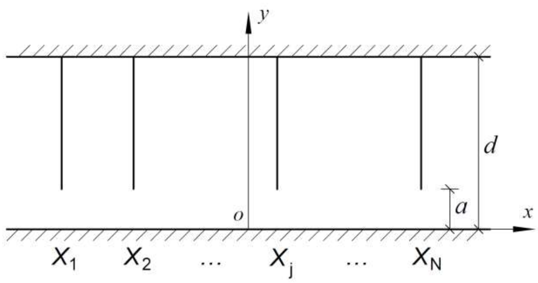


Fig. 1 Definition of the problem

For the problem of monochromatic wave diffraction from vertical uniform bodies, the time factor and the vertical eigen function may be separated out. Thus, the velocity potential in a water depth $h$ can be written as

$$\Phi(x,y,z,t) = \mathrm{Re}[\phi(x,y)\frac{\cosh k(z+h)}{\cosh kh}\mathrm{e}^{-i\omega t}]\,. \tag{1}$$

The horizontal complex potential $\phi(x,y)$ satisfies the Helmholtz equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+k^2\phi=0 \tag{2}$$

where $k$ is the wave number, which satisfies the dispersion relationship with wave frequency $\omega$.

We divide the wave potential into the incident and diffraction potentials as

$$\phi(x,y)=\phi_I(x,y)+\phi_d(x,y)\,. \tag{3}$$

The incident wave with an amplitude of $A$, can be written as

$$\phi_I = -\frac{igA}{\omega} e^{ikx} \tag{4}$$

The diffraction potential is to be determined, and satisfies the following conditions:

$$\frac{\partial \phi_d}{\partial n} = -\frac{\partial \phi_I}{\partial n}\ , \tag{5}$$

on the body surface $S_B$;

$$\frac{\partial \phi_d}{\partial y} = 0 \tag{6}$$

on the channel walls $y$=0 and $y$=$d$, and the radiation conditions at the upper and the lee sides of the channel.

At the upper side of the channel, the diffraction potential can be written as

$$\phi_d = -\frac{igA}{\omega}[\mathrm{A}_0^- e^{-ikx} + \sum_{j=1}^{\infty} \mathrm{A}_j^- \mathrm{e}^{-\kappa_j x} \cos\lambda_j y], \quad x < X_1 \tag{7}$$

and at the lee side of the channel the diffraction potential can be written as

$$\phi_d = -\frac{igA}{\omega}[\mathrm{A}_0^+ e^{ikx} + \sum_{j=1}^{\infty} \mathrm{A}_j^+ \mathrm{e}^{\kappa_j x} \cos\lambda_j y], \quad x > X_N \tag{8}$$

where the eigen values for the transverse oscillations are

$$\lambda_j = \frac{j\pi}{d}, \quad j = 0, 1, 2, ... \tag{9}$$

Substitution of the diffraction potentials into Helmholtz equation, it yields

$$\kappa_j^{\ 2} - \lambda_j^2 + k^2 = 0\,. \tag{10}$$

Correspondingly, the eigen value for the longitudinal oscillation is

$$\kappa_j = \sqrt{\lambda_j^2 - k^2}, \quad j = 1, 2, ... \tag{11}$$

When the channel is wide, or wave number is high, wave number $k$ may be larger than some transverse eigen values $\lambda_j 's$. Thus, we accordingly define those longitudinal eigen values as

$$\kappa_j = \pm i\sqrt{k^2 - \lambda_j^2} = \pm ik_j, \quad j = 0, 1, ..., J$$

when $k$ is in the range $(J\pi/d, (J+1)\pi/d)$. At a far distance from the barriers, the diffraction potentials can be approximated as

$$\phi_d = -\frac{igA}{\omega}\sum_{j=0}^{J} \mathrm{A}_j^{\pm} \mathrm{e}^{\pm ik_j x} \cos\lambda_j y, \quad x \to \pm\infty\ . \tag{12}$$

## 3 BEM for thin vertical barriers in a Channel

With applying the Channel Green's function to the Second Green Theorem, the integration equation for diffraction waves can be obtained as follows (Linton and Evans, 1992):

$$\alpha\phi_d(x_0) = \int_{S_B} [\frac{\partial G}{\partial n}\phi_d(x) - \frac{\partial \phi_d(x)}{\partial n} G] ds \tag{13}$$

where α is the solid angle of fluid domain and **n** is the normal direction of the body surface directing out off the fluid domain.

For barriers with zero thickness, the above integral equation can be written as (Linton and McIver, 2001)

$$\alpha\phi_d(x_0) = \int_{S_B} \frac{\partial G(\mathrm{x}, \mathrm{x}_0)}{\partial n^+} \Delta\phi_d(x) dS \tag{14}$$

where $\Delta\phi_d(x) = \phi_d(x^+) - \phi_d(x^-)$ is the difference of diffraction potentials on the positive and the negative sides of the barrier. Allocating the source point $\mathrm{x}_0$ on the body surface and taking normal derivative of the integral equation at the source point, it yields

$$-\alpha\frac{\partial \phi_I(\mathrm{x}_0)}{\partial n(x_0)} = \mathrm{HD}\int_{S_B} \frac{\partial^2 G(\mathrm{x}, \mathrm{x}_0)}{\partial n^+(x)\partial n(x_0)} \Delta\phi_d(x) ds \tag{15}$$

with application of body surface condition, where HD represents the Hadamard integration. With this integral equation, the difference of the diffraction potentials on barrier's two sides can be determined.

At a large distance from bodies in a channel, the channel Green's function can be approximated as (Linton and Evans, 1992)

$$G(\mathbf{x}, \mathbf{x}_0) = \sum_{j=0}^{J} G_{Sj}^{\pm}(\mathbf{x}_0) G_{Fj}^{\pm}(\mathbf{x}), \quad x \to \pm\infty \tag{16}$$

where

$$G_{Sj}^{\pm}(\mathrm{x}_0) = -\frac{i}{2d}\frac{\varepsilon_j}{k_j} e^{\mp ik_j x_0} \cos\lambda_j y_0$$

$$G_{Fj}^{\pm}(\mathbf{x}) = e^{\pm ik_j x} \cos\lambda_j y$$

With substitution of the Green's function into Eq. (14), the amplitude of each propagation wave mode of reflection and transmission waves can be computed by

$$A_j^{\pm}=\frac{\varepsilon_j\omega}{2dk_j gA}\int_{S_B}\frac{\partial G_{Fj}^{\mp}(\mathbf{x})}{\partial n^{+}(\mathbf{x})}\Delta\phi_d(\mathbf{x})\,ds\,. \qquad (17)$$

## 4 Reflection and Transmission Coefficients in Channels

The wave energy flux across a channel section may be obtained by the following integration over a channel section

$$F=-\frac{\omega^2\rho}{2gk}C_g\int_0^d \mathrm{Im}[\phi\frac{\partial\phi^*}{\partial x}]\mathrm{d}y \qquad (18)$$

With substitute of the incident and the diffraction potentials into Eq. 18, the wave energy flux across a section at the upper side of the channel may be separated into the fluxes of incident and diffraction waves as

$$F=F_I-F_R \qquad (19)$$

where

$$F_I=\frac{1}{2}g\rho A^2C_g d \qquad (20)$$

$$F_R=F_I\sum_{j=0}^{J}\frac{k_j}{2k}\left|A_j^-\right|^2 \qquad (21)$$

and $c_g$ is the wave group velocity. The wave energy flux across the lee side of the channel is

$$F_T=F_I(\left|1+A_0^+\right|^2+\sum_{j=1}^{J}\frac{k_j}{2k}\left|A_j^+\right|^2) \qquad (22)$$

By the energy conservation law in a described fluid domain, the following relation can be obtained as

$$F_R/F_I+F_T/F_I=1 \qquad (23)$$

This equation is the same as that derived by Srokosz (1980) with using Green theorem.

The reflection and transmission coefficients can be defined by the ratios of the energy fluxes of reflection and transmission waves with that of incident waves as

$$K_R=\sqrt{F_R/F_I} \qquad (24)$$

and

$$K_T=\sqrt{F_T/F_I}\quad . \qquad (25)$$

## 5 Numerical Examples

The hyper singular integral equation (Eq. 15) is discretized by constant panels, and a BEM model is set up. The model is validated by energy conservation relation between incident, reflection and transmission waves, and Evans & Porter's (2015) small gap approximation results. In follows, only the transmission coefficient is plotted for brevity. With this numerical model, computations are carried out to examine the influence of barrier's number, channel width, gap widths, and barrier arrangements on wave transmission in channels.

Fig. 2 shows the wave transmission coefficients for channels with different number of barriers. It can be seen that with the increase of barrier's number, the transmission coefficients have more peaks at low wave frequencies. The number of ET frequencies is one less than the number of barriers, and the fundamental ET frequencies is related with $k(X_N-X_1)$. At higher wave number ($kd$>2), more peaks will occur (as shown in Fig. 3).

Fig. 3 is the results for channels of different widths. 4 barriers are arranged in the channel with same interval $b$ and gap width $a$. It can be seen that ET frequency decreases with the increase of channel width. At higher frequency more peaks appear with the generation of auxiliary propagation waves.

Fig. 4 shows the results for channels with same width but different gap sizes. It can be seen that gap width also has evident influence on ET frequencies. With the increase of gap width, the ET frequencies increase, and the high transmission range broadens either.

Fig. 5 shows the results for channels with different arrangements of barriers. In all the computation, 4 barriers are distributed in the channel, with gap width of $a/d$=0.1. Barrier locations for the four computation cases are listed in Table 1. It can be seen that the arrangement of the barriers has little influence on the fundamental ET frequency, as $(X_4-X_1)$ are the same, but has evident influence on the second and the third ET frequencies. It means that with careful design of barrier arrangement the high transmission range can be narrowed.

## 6 Summary

A hyper singular BEM model is set up for wave diffraction from thin vertical barriers in channels. With application of the model, numerical examinations are carried out on the influence of barrier's number, channel width, gap width, barrier interval and barrier arrangement on wave transmission. It is found that:

(a) all of those factors have influence on wave ET frequencies,

(b) at high frequency auxiliary propagation waves will induce high transmission either,

(c) the fundamental ET frequency is dominated by the interval between the first and the last barriers, but high transmission frequency range can be narrowed by barrier arrangement.


## Acknowledgments

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## References


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Table 1 Barrier positions for computation cases

| Cases | $X_1$/d | $X_2$/d | $X_3$/d | $X_4$/d |
|---|---|---|---|---|
| Case 1 | -1.5 | -0.5 | 0.5 | 1.5 |
| Case 2 | -1.5 | -0.75 | -0.75 | 1.5 |
| Case 3 | -1.5 | 0.0 | 1.0 | 1.5 |
| Case 4 | -1.5 | 0.0 | 0.8 | 1.5 |

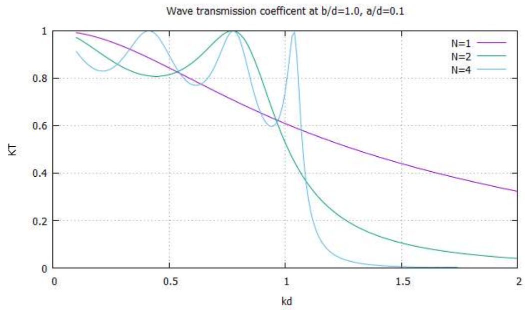


Fig. 2 Wave transmission coefficients for a channel with different number of barriers

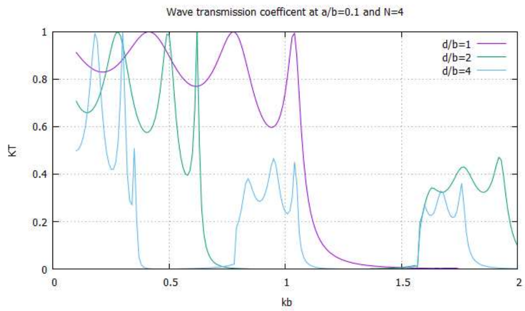


Fig. 3 Wave transmission coefficients for channels with different widths but the same gap width and barrier interval

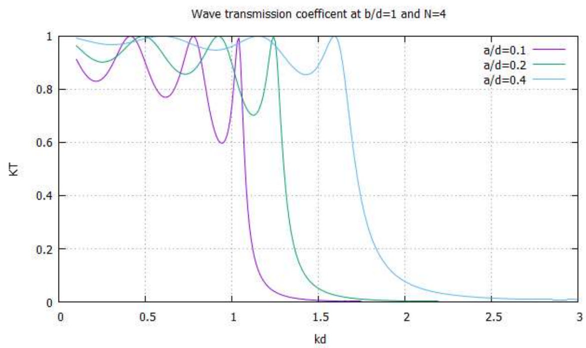


Fig. 4 Wave transmission coefficients for channels with same width and barrier intervals but different gap sizes

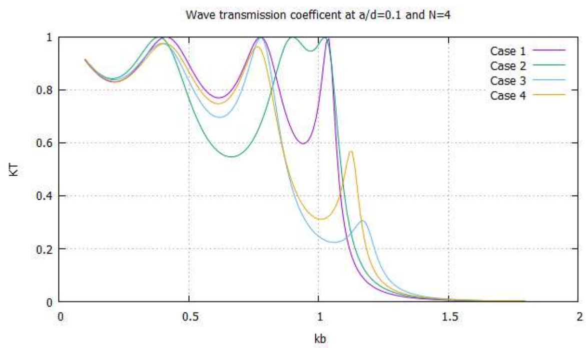


Fig. 5 Wave transmission coefficients for channels with different arrangements of barriers